Causality 4 Climate

Team CoCaLa (Copenhagen Causality Lab), University of Copenhagen

December 14th, 2019, NeurIPS Competition Track Day 2



Team: 4 PhDs and 2 Postdocs from the Copenhagen Causality Lab (CoCaLa).



CoCaLa is a group of 12 people in Statistics at the University of Copenhagen doing research in causal inference.

Causal challenges in dynamical systems are different to those on iid data.

- With no unobserved process, it is well-known that structure is identifiable.
- Here: Different graphs cannot be Markov equivalent.
- This is because time helps to orient the edges.



(Meek 2014; Mogensen and Hansen 2018)

Overview of final methods

File	Description	Edgescore	Datasets
ridge.py	Ridge regression $\Delta X_t \approx f(X_{t-1})$. Bootstrap sampling, aggregation by quantiles.	Absolute values of regression coefficients	Climate data
varvar.py	OLS regression $X_t \approx f(X_{t-1,,t-lags})$ or iterative Lasso $X_t - f(X_{t-1}) \approx f(X_{t-2})$. Bootstrap sampling, random number of lags.	Absolute values of regression coefficients	Weather data, VAR models
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• We also tried: PC-type algorithms, neural-networks with input-perturbations, state space models, residual entropy estimation, ...

Ridge regression and bootstrap quantiling

Ridge and bootstrap quantiling

Target $Y_t = X_t - X_{t-1}$, **Regressor** X_{t-1} **for** $i \in \{1, ..., N\}$ **do**: Draw bootstrap samples Y^i and X^i Perform **Ridge** regression $Y^i \sim X^i$, obtain estimate $\hat{A}_i \in \mathbb{R}^{d \times d}$. Collapse $\hat{A}_1, ..., \hat{A}_N$ into one $\hat{A} \in \mathbb{R}^{d \times d}$ by entrywise taking the q^{th} quantile. **return** abs (\hat{A}) as scores for causal links

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Linear regression simulation study

- Choosing *q* large corresponds to searching for largest *local* effect.
- Choosing *q* small corresponds to large *minimal* effect.



What about covariance?

• Consider iid observations from the well-known linear acyclic model

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_d \end{bmatrix} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ b_{21} & 0 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ b_{d1} & b_{d2} & \dots & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_d \end{bmatrix} + \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_d \end{bmatrix}$$

where $E \sim \mathcal{N}(0, \sigma^2 I)$.

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- Compare the AUC when scoring edges $X_i \rightarrow X_j$ either by the
 - (a) absolute regression coefficients $|\widehat{b}_{i \to j}|$, or by the
 - (b) corresponding absolute t-test statistics $|\hat{t}_{i\to j}|$.

Equal error variance



Equal error variance



 $d = 50, n = 200, p_0 = 0.75, 100$ repetitions

From absolute coefficients to t-test statistics

•
$$|\widehat{b}_{i \to j}|$$

•
$$|\widehat{t}_{i \to j}| = |\widehat{b}_{i \to j}| \sqrt{\frac{\widehat{\operatorname{var}}(X_i)}{\widehat{\operatorname{var}}(X_j)}} \sqrt{\frac{(n-2)}{(1-\widehat{\rho}_{X_i,X_j}^2)}}$$

(pairwise regression)

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 $\widehat{}$

•
$$|\widehat{t}_{i \to j}| = |\widehat{b}_{i \to j}| \frac{\operatorname{sre}(X_i | X_{\setminus j})}{\widehat{\operatorname{sre}}(X_j | X_{\setminus j})} \sqrt{\frac{(n-d)}{(1 - \widehat{\rho}_{X_i, X_j | X_{\setminus \{i, j\}}}^2)}}$$

(multivariate regression)

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- Consider, as before, X = BX + E with $E \sim \mathcal{N}(0, \text{diag}(\sigma_1^2, \dots, \sigma_d^2))$.
- This time (somewhat artificially) ensure equal marginal variances

$$\operatorname{var}(X_i) = c, \ i = 1, \ldots, d,$$

by rescaling rows of B and the σ_i^2 appropriately.

Equal marginal variance



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• This time (somewhat artificially) ensure decreasing marginal variances,

 $\forall i < j :$ var $(X_i) \ge$ var (X_j) ,

by rescaling rows of B and the σ_i^2 appropriately.

Decreasing marginal variance



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- For inferring causal structures, we argue that it may not be sufficient to consider the AUC.

• We consider two graphs with binary adjacency matrices:

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• Only difference: Flipped edge between the two drivers.

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- The AUC (as calculated in the competition) increases quickly as p grows.
- The number of incorrectly inferred interventional distributions (SID, see Peters and Bühlmann 2015) is 2(p 1).
- The following distributions are incorrectly inferred (for fixed $c \in \mathbb{R}$):

$$\mathbb{P}_{D_A}^{\operatorname{do}(D_B=c)}, \qquad \mathbb{P}_{D_B}^{\operatorname{do}(D_A=c)}, \qquad \mathbb{P}_{D_B}^{\operatorname{do}(D_A=c)}, \ \mathbb{P}_{Y_k}^{\operatorname{do}(D_B=c)}, \qquad \mathbb{P}_{Y_k}^{\operatorname{do}(D_B=c)}$$

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- The distribution of Y under $do(D_B = c)$ depends on the graph it is calculated under.

Estimated densities of $\mathbb{P}_Y^{\operatorname{do}(D_B=c)}$



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• The estimated distribution can be arbitrarily wrong with just a single edge flipped.

- Open question: What does it mean for two causal systems to be 'close' to one another?
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- Take-home message: Inferring causal structures is difficult, but:

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- For inferring causal structures, we argue that it may not be sufficient to consider the AUC.
- Take-home message: Inferring causal structures is difficult, but:
 - If you are interested in an overall structure, the AUC is a good measure.
 - If you care about interventional distributions, it is not sufficient to consider only the AUC.

Final remarks

- Linear regression can beat 'causality tailored' and non-linear approaches.
- When is covariance a good indicator of causality?
- The task is only as causal as the score function!

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Thanks to the organizers!

CoCaLa: https://www.math.ku.dk/english/research/spt/cocala/ Code: https://github.com/sweichwald/CoCaLa-CauseMe-NeurIPS-competition

X Thanks also for accomodating our telepresence which saved \sim 28500 kg in CO₂ emissions.